Notes: 10.4 Matrices

matrix: a rectangular array of terms called elements

array: an arrangement of data

row matrix: has only one row

column matrix: has only one column

<u>m x n matrix</u>: has "m" rows and "n" columns *(dimensions = m by n)*

example: 3 by 2 matrix → $\begin{bmatrix} 5 & -6 \\ 2 & 11 \\ 7 & -1 \end{bmatrix}$

example: 2 by 3 matrix → $\begin{bmatrix} -2 & 5 & 0 \\ 6 & -1 & 13 \end{bmatrix}$

square matrix: has same number of rows and columns

nth order: refers to square matrices

3rd order → 3x3 matrix

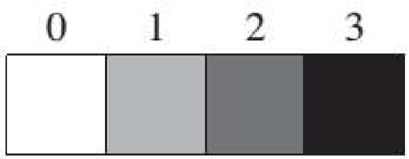
4th order → 4x4 matrix

scalar: a constant value that multiplies all values inside a matrix

example: $$3\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -12 & 15 \end{bmatrix}$$

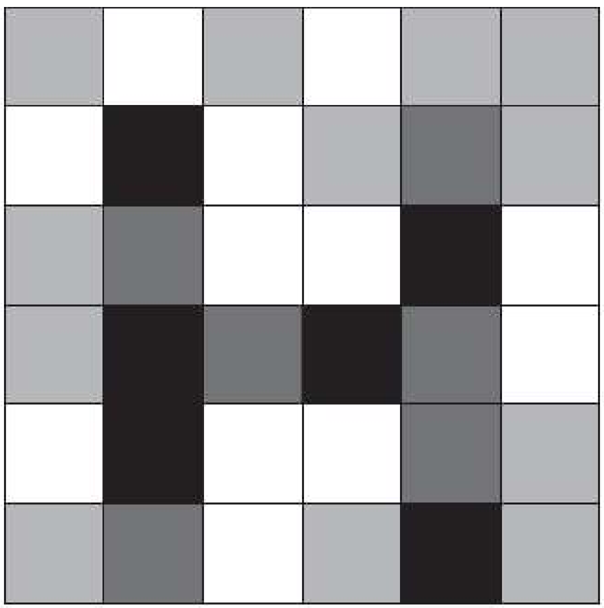
EXAMPLE: Computer Graphics

Digital Images A four-level gray scale is shown below.



(a) Use the gray scale to find a 6 × 6 matrix that digitally represents the image in the figure.

One important use of matrices is in the digital representation of images. A digital camera or a scanner converts an image into a matrix by dividing the image into a rectangular array of elements called pixels. Each pixel is assigned a value that represents the color, brightness, or some other feature of that location.

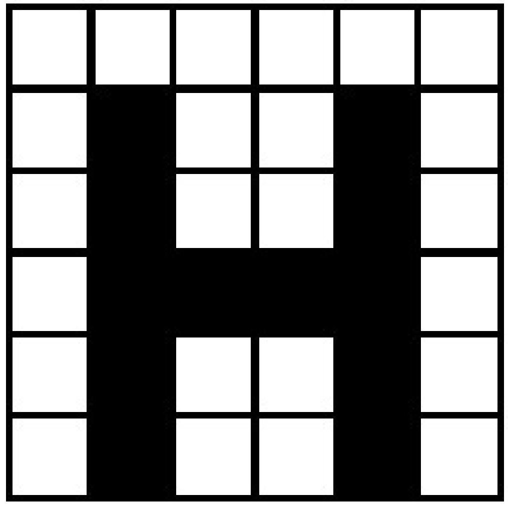


$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 & 3 & 0 \\ 1 & 3 & 2 & 3 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 & 3 & 1 \end{bmatrix}$$

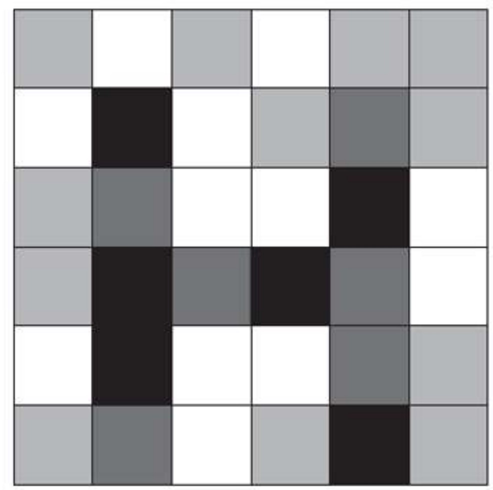
FYI: do not write these notes, just read them

EXAMPLE:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 3 & 3 & 3 & 3 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 & 3 & 0 \\ 1 & 3 & 2 & 3 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 & 3 & 1 \end{bmatrix}$$



FYI: do not write these notes, just read them

When multiplying matrices:

The number of *columns in the 1st matrix* must be equal to the *number of rows in the 2nd matrix* in order to find a solution.

$$\begin{bmatrix} a & b \end{bmatrix} \bullet \begin{bmatrix} c & d \\ e & f \end{bmatrix} = \begin{bmatrix} ac+be & ad+bf \end{bmatrix}$$

a single value — Apply row 1 to column 1

a single value — **Apply row 1 to column 2**

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Example for notes:

$$X = \begin{bmatrix} 4 & 1 \\ -2 & 6 \end{bmatrix} \qquad Z = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$XZ = \begin{bmatrix} 4(-1) + 1(0) & 4(3) + 1(-2) \\ -2(-1) + 6(0) & -2(3) + 6(-2) \end{bmatrix}$$

Set up

$$= \begin{bmatrix} -4 & 10 \\ 2 & -18 \end{bmatrix}$$

Solve

9–16 ■ Matrix Operations Perform the matrix operation, or if it is impossible, explain why.

10.4 #9

$$\begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$$

10.4 #10

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 2 \end{bmatrix}$$

9–16 ■ Matrix Operations Perform the matrix operation, or if it is impossible, explain why.

10.4 #11

$$\text{scalar } 3\begin{bmatrix} 1 & 2 \\ 4 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 12 & -3 \\ 3 & 0 \end{bmatrix}$$

10.4 #12

$$2\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

not possible
dimension
mismatch

9–16 ■ Matrix Operations Perform the matrix operation, or if it is impossible, explain why.

10.4 #13

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix}$$

not possible

dimension mismatch

9–16 ■ Matrix Operations Perform the matrix operation, or if it is impossible, explain why.

10.4 #14

$2(1) \quad 1(3) \quad 2(-2)$

$$\begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 2+3+-4 & -4+6+0 \\ 6+9+-8 & -12+18+0 \end{bmatrix}$$

apply each row → to each column (find products, then add)

$$= \begin{bmatrix} 1 & 2 \\ 7 & 6 \end{bmatrix}$$